



Brief article

Individual differences in nonverbal number skills predict math anxiety



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ABSTRACT

Math anxiety (MA) involves negative affect and tension when solving mathematical problems, with potentially life-long consequences. MA has been hypothesized to be a consequence of negative learning experiences and cognitive predispositions. Recent research indicates genetic and neurophysiological links, suggesting that MA stems from a basic level deficiency in symbolic numerical processing. However, the contribution of evolutionary ancient purely nonverbal processes is not fully understood. Here we show that the roots of MA may go beyond symbolic numbers. We demonstrate that MA is correlated with precision of the Approximate Number System (ANS). Individuals high in MA have poorer ANS functioning than those low in MA. This correlation remains significant when controlling for other forms of anxiety and for cognitive variables. We show that MA mediates the documented correlation between ANS precision and math performance, both with ANS and with math performance as independent variable in the mediation model. In light of our results, we discuss the possibility that MA has deep roots, stemming from a non-verbal number processing deficiency. The findings provide new evidence advancing the theoretical understanding of the developmental etiology of MA.

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1. Introduction

In modern society people highly depend on numerical abilities to make well-informed choices and decisions (Paulos, 1988). One factor with great impact on these abilities is math anxiety (MA). MA has both direct effects, in terms of poor math performance, and indirect life-long effects, in terms of education and career path choice (Ashcraft, 2002; Hembree, 1990). MA, defined as "...feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations." (Richardson & Suinn, 1972, p. 551), is negatively associated with performance on arithmetic problem solving (Fig. 1a, path B)¹ (Ashcraft & Faust, 1994). Ashcraft and Faust (1994), in keeping with processing efficiency theory (Eysenck & Calvo, 1992), have suggested that this occurs because anxiety during arithmetic problem solving takes up working memory (WM) resources in individuals high in MA. According to this account there are no competence differences in arithmetic processing per se between high and low MA individuals and the negative

effect of MA on performance will only occur for mathematical tasks with a certain amount of cognitive load.

MA is positively related both to test anxiety and other types of anxiety, such as trait-anxiety (an enduring predisposition to feel wide-ranging stress, worry, and discomfort) and state-anxiety (susceptibility to arousal induced temporarily by various situations perceived as dangerous), but research suggests that the phenomenon is independent of and not reducible to these constructs (Ashcraft, 2002). Further, MA persists from childhood to adulthood, probably reinforced by avoidance behavior in terms of shunning math courses in high school and college (Hembree, 1990).

Explanations of the development of MA involve exposure to failure in mathematics, negative attitudes transferred by teachers, and cognitive predispositions. Neurologically, MA invokes affective activations in the pain and fear networks in the brain (Lyons & Beilock, 2012; Young, Wu, & Menon, 2012). Recently, Wang et al. (2014) showed that genetic factors account for 40% of the variation in MA, suggesting the existence of biological pathways.

Number symbols and mathematics are fairly recent cultural inventions that unlikely have had time to make genetic imprints. However, the ability to process and manipulate symbolic numbers is preceded, both ontogenetically and phylogenetically, by an ability to represent numerical magnitudes nonverbally without the aid of symbols. This "number sense" (Dehaene, 1992) is thought to be supported by a core cognitive system – The Approximate Number System (ANS) – that has been documented in infants, adults, and

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¹ Fig. 1a is intended to illustrate a pattern of observed relations between MA, math performance, and ANS acuity. In contrast to a mediation model, the links lack any direction to emphasize that the figure is neutral with respect to the causal direction of these relations.

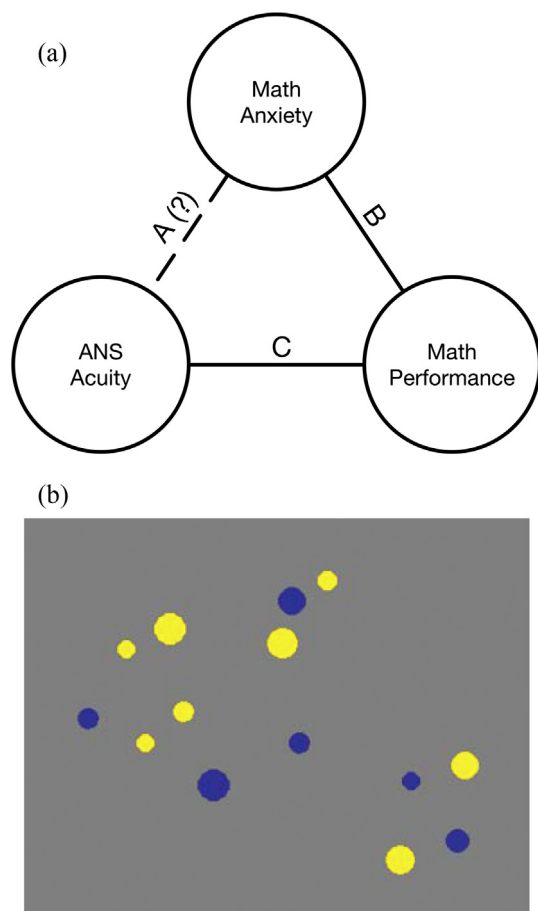


Fig. 1. Panel (a) overview of the study aim. Correlational paths B and C are well-documented empirically. The aim was to investigate the association in path A and potential consequences for the other associations. Note that the figure shows a pattern of observed correlations and is thus neutral with respect to the causal direction of the relations. Panel (b) illustration of a typical ANS task. Participants are briefly presented with two sets of dots and judge, intuitively without counting, which is the more numerous set.

non-human animals alike (Feigenson, Dehaene, & Spelke, 2004). Typical tasks measuring the efficiency of this system involve rapid intuitive judgments of the relative numerosity of sets of objects (Fig. 1b).

There are substantial individual differences in the precision, or *acuity*, with which the ANS represents numerosity (Halberda & Feigenson, 2008) and a documented positive association between ANS acuity and math performance (Fig. 1a, path C) (Chen & Li, 2014; Halberda, Mazocco, & Feigenson, 2008; Lindskog, Winman, Juslin, & Poom, 2013; Price, Palmer, Battista, & Ansari, 2012). Individuals with a more acute ANS show better performance on arithmetic tasks (Halberda et al., 2008) and children suffering from dyscalculia exhibit an impairment in ANS acuity (Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010). This observed association between ANS acuity and math performance could indicate a causal link from ANS to math performance. It has been proposed that the ANS lays the foundation for the development of symbolic math (e.g., Gilmore, McCarthy, & Spelke, 2007). A finding in line with this position is that pre-verbal number sense (ANS) in 6-month old infants predicts math ability three years later (Starr, Libertus, & Brannon, 2013). Further, in experimental studies, training with non-symbolic addition and subtraction has also been claimed to enhance math performance (Park & Brannon, 2013, 2014).

Other research however suggests a causal link in the reversed direction, by which math education or familiarization with symbolic numbers modifies and sharpens the ANS. It has, in accord with this interpretation, been shown that among people brought up in Western cultural context, schooled adults with formal math education have better ANS acuity than unschooled adults (Nys et al., 2013). Research on children likewise suggests that whereas ANS acuity fails to predict later performance with symbolic numbers, symbolic number skills predict later ANS acuity (Matejko & Ansari, 2016; Mussolin, Nys, Content, & Leybaert, 2014).

Math performance has been measured with a wide variety of dependent variables, ranging from conceptual math knowledge to SAT scores or direct measures of computational arithmetic performance. Across studies, using various measures of mathematical performance and ANS acuity precision, an average zero order correlation of 0.23 has been estimated (Chen & Li, 2014). Here, we use a measure of arithmetic fluency to measure math performance, which in previous research has been documented to correlate with ANS precision (Gebuis & van der Smagt, 2011; Lindskog, Winman, & Juslin, 2014; Lindskog et al., 2013; Lourenco, Bonny, Fernandez, & Rao, 2012).

Previous research has controlled for various cognitive factors that might explain the relation between ANS acuity and math performance. Lyons and Beilock (Lyons & Beilock, 2011), for example, showed with an approach similar to the present study that the relation was mediated by participants' symbolic number-ordering ability. Much of cognitive research, however, tends to focus on cognitive processes in isolation without addressing affective variables. Consequently, components such as MA have largely been overlooked as mediating variables.

Recent research (Maloney, Ansari, & Fugelsang, 2011; Maloney, Risko, Ansari, & Fugelsang, 2010; Núñez-Peña & Suárez-Pellicioni, 2014) suggests that MA may be related not only to mathematical problem solving, but also to more fundamental symbolic numeric abilities such as simple enumeration of objects and representation of symbolic numbers. High MA individuals also show less efficient neural processing in basic numerical tasks (Artemenko, Daroczy, & Nuerk, 2015). This suggests an origin of MA in the most fundamental nonverbal level of number processing (i.e., ANS acuity). We consequently hypothesized that MA would be negatively related both to math performance, involving symbolic numeric processing, and to ANS acuity (Fig. 1a, path A). On the account of the notion that MA evokes tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems (Richardson & Suinn, 1972), that is in situations including symbolic numerical content, the latter association is not expected. Associations in paths B and C (Fig. 1a) are well documented. The study aim was to probe for an association corresponding to path A in Fig. 1a. Such an association would indicate that MA stems from a very basic nonverbal number processing deficiency and that MA may account for the documented relation between ANS acuity and math performance (path C). Because there is still a debate in the literature concerning the causal direction of the relation between ANS acuity and math performance, we investigated the relations in Fig. 1a both by considering ANS and when considering math performance as an independent variable.

2. Method

2.1. Participants

Eighty-eight (58 women) undergraduates ($M_{\text{age}} = 24.3$ years, $SD = 5.5$) took part. In the final sample used in the analysis below 8 participants were excluded due to missing data points because of apparatus or experimenter error. One participant not performing

above chance was considered being an outlier and was excluded. Participants received a movie voucher or course credits as compensation. Recent research has suggested that the effect size for the relation between ANS and performance in various math task is of a small to medium size (Chen & Li, 2014). In the current study we set sample size prior to data collection and motivated by the desire to have an approximate power of 0.8 to detect a medium effect size ($r = 0.3$) according to Cohen (1992).

2.2. Materials and procedure

ANS acuity, math performance, math anxiety, test anxiety, state- and trait-anxiety, number/letter scanning speed and general cognitive functioning were measured with a battery of tests described below. Tasks were carried out in the order they appear below.² A session took approximately 80 min.

2.3. Number measures

2.3.1. ANS acuity

The task was based on Halberda et al. (2008). On 300 trials, participants were presented with an array of spatially intermixed blue³ and yellow dots on a grey background (see Fig. 1b for an example). They decided which set of dots was more numerous by pressing a color-coded key. Stimuli were presented on a computer screen using Psychtoolbox (Brainard, 1997) in MATLAB. Presentation time (300 ms) was brief in order to preclude serial counting. There was no time constraint enforced on participants' responses. Once a response had been given, participants initiated the next trial. Dots varied randomly in size, with radii subtending visual angles varying between 0.5° and 0.9°. Entire array of blue and yellow dots covered approximately 13 × 13 visual degrees. Half of the trials had blue and half had yellow as the more numerous color. The ratio of the two numbers of dots varied over four levels (3:4, 5:6, 7:8, 9:10), with the total number of dots varying between 11 and 30. A quarter of the trials consisted of each ratio. To minimize the possibility of perceptual cue use other than numerosity, arrays were matched for total area on half of the trials and average dot-size on the other half (see e.g., Halberda et al., 2008). Previous research has indicated that 300 trials is necessary to reach an approximate reliability of 0.75 (Lindskog et al., 2013). Proportion of correct trials was used as the dependent variable (Inglis & Gilmore, 2014; Lindskog et al., 2013).

2.3.2. Math performance

We measured math performance with a test of arithmetic fluency adapted from Gebuis and van der Smagt (2011). Participants carried out a computerized task consisting of four timed (150 s) sets of arithmetic problems (addition, subtraction, multiplication, and division). They were instructed to correctly complete as many problems as possible, within the 150 s time limit. Problems within each set became increasingly more difficult by adding more digits in the problem and/or by requiring borrowing or carrying. For example the first three problems in the addition and multiplication sets were 2 + 7, 12 + 9, and 38 + 17, and 2 · 3, 3 · 6, and 4 · 7, respectively. All participants carried out the problems within each set in

the same order and they were given no feedback about the correctness of their response. The dependent measure was the total number of correctly solved problems over the four sets of tasks.

2.4. Affective measures

2.4.1. Math anxiety

The test by Hopko (2003) was used. The test is a short version of the Math Anxiety Rating Scale (Richardson & Suinn, 1972). The test measures a combination of two factors of math anxiety, which address anxiety when learning math (learning math anxiety, LMA) and when being evaluated at math (evaluation math anxiety, EMA), respectively. Previous research has reported strong reliability coefficients for both the LMA (Cronbach $\alpha = 0.87$) and EMA (Cronbach $\alpha = 0.85$) sub-scales (Hopko, 2003).

2.4.2. State-, trait-, and test-anxiety

State- and trait anxiety were measured with a short version (STAI-6: Fioravanti-Bastos, Cheniaux, & Landeira-Fernandez, 2011) of the State-Trait Inventory (STAI: Spielberg, Gorsuch, Lushene, Vagg, & Jacobs, 1983), which shows good reliability for both the state (Cronbach $\alpha = 0.75$) and trait (Cronbach $\alpha = 0.73$) sub-scales (Fioravanti-Bastos et al., 2011). General Test Anxiety was measured with a short version (TAI-5: Taylor & Deane, 2002) of the Test Anxiety Inventory (TAI: Spielberg et al., 1980), exhibiting good reliability (Cronbach $\alpha = 0.87$).

2.5. Cognitive measures

2.5.1. Scanning speed

We developed a computerized test of individuals' speed of comparing two strings of capital letters or Arabic numerals to measure low-level processing, or scanning speed, of numbers and letters. The strings were presented side by side on the computer screen and participants decided if the strings were identical or different. Strings were 9 characters long. Half of the pairs of strings were identical (e.g., 8 3 7 1 0 9 3 6 7 vs. 8 3 7 1 0 9 3 6 7), on the other half pairs differed by one character (e.g., Q L B D P A X F V vs. Q L C D P A X F V). Both (number/letter scanning) tests were time-pressured, with a two-minute limit. We used the number of correct responses as the dependent measure of scanning speed for number and letter scanning, respectively.

2.5.2. General intelligence

High-level cognitive functioning was measured with a subset of Raven's progressive matrices (Raven, Raven, & Court, 1998) based on Stanovich and West (1998). This test is generally used as a proxy to fluid intelligence (Stanovich & West, 1998). During the computerized task, participants completed two practice items before completing 18 of the test items (items 13 through 30) with a 15 min time limit. Stanovich and West (1998) showed that this version of Raven's progressive matrices, which eliminates the 12 easiest and the 6 most difficult problems, results in a time-efficient and reasonably reliable measure of fluid intelligence (Spearman-Brown corrected split-half reliability of 0.69). Participants were instructed to try to complete all 18 items within the time limit. We used the total number of correctly solved items as the dependent measure.

3. Results

Correlations between all variables together with descriptive statistics for each variable separately can be seen in Table 1. The anxiety measures were positively correlated. Higher general intelligence and lower test anxiety, accompanied better performance

² Task order was held constant for two reasons. First, we wanted to introduce as few cues as possible about numerical contents, and thus possible feelings of anxiety for those high in MA, before the ANS task was administered. Second, we wanted to introduce as little noise as possible into our measurements as to not confound individual differences with task order. Due to this, we cannot exclude the possibility that the particular order in which participants perform these tasks may have affected the results. More specifically, it is possible that the MA ↔ MATH relation may have been inflated by this order. However, based on our previous observations we do not have reason to believe that order effects play a significant role here.

³ For interpretation of color in Fig. 1, the reader is referred to the web version of this article.

Table 1
Pearson correlations between all measured variables and descriptive statistics for all measured variables.

Measure	Measure									Descriptive statistics			
	1	2	3	4	5	6	7	8	9	M	SD	Skew	Kurtosis
1. Math performance		0.31*	-0.43**	0.14	-0.07	-0.21	0.36**	0.15	0.52**	42.5	12.7	0.52	-0.41
2. ANS acuity			-0.35*	0.05	0.02	-0.22	0.18	0.14	0.31*	0.69	0.05	-0.11	-0.44
3. Math anxiety				0.40**	0.47**	0.61**	-0.13	0.02	-0.30*	17.4	11.5	0.65	-0.54
4. State anxiety					0.49**	0.38**	0.10	0.11	0.13	10.9	4.0	0.92	0.46
5. Trait anxiety						0.39**	0.04	0.07	-0.01	13.0	3.8	0.44	-0.51
6. Test anxiety							-0.16	0.01	-0.18	9.0	3.0	0.67	-0.12
7. Number scanning speed								0.51**	0.32*	25.8	6.1	-0.40	0.78
8. Letter scanning speed									0.15	28.1	6.3	0.51	0.23
9. General intelligence										8.4	3.9	0.25	-0.68

Note. N = 79 for all correlations.

* p < 0.005.

** p < 0.001.

on the math test. Number scanning speed was related to math performance, letter scanning speed and IQ.

More central to our research question, we first replicated the finding that better ANS acuity was related to math performance. The zero order correlation with math performance was $r(77) = 0.31$ which is in the upper range of what is usually found between ANS acuity and various math tests (Chen & Li, 2014).⁴

Second, we also found that MA correlated negatively with math performance, $r(77) = -0.43$, corroborating that MA is inversely related to symbolic numeric calculation abilities. Controlling for other measures by regressing math performance on these showed that the partial correlation between MA and math performance remained statistically significant (Table 2, section i).

Third, we also found a negative correlation between ANS acuity and MA $r(77) = -0.35$. Importantly, this correlation remained significant when controlling for all other measures (Table 2, section ii) by regressing MA on these. Thus, the relationship between ANS acuity and MA does not exist due to either general cognitive functioning, other types of anxiety or low-level information scanning speed. This shows the robustness of this association and that ANS acuity and MA share unique variance.

We here primarily conceptualize ANS as an independent, exogenous variable. Regressing this variable on the other covariate factors in the reversed direction, as a criterion variable, reveals that with this analysis only MA shares unique predictive variance with ANS acuity (Table 2, section iii). It is thus harder to find predictors of ANS acuity than it is to find significant predictors either of math performance or math anxiety. This is consistent with the view of ANS as a more or less hard-wired, non-malleable precursor variable.

3.1.1. Mediation analysis

We used a mediation analysis with bootstrapping (Preacher & Hayes, 2008) to explore the mediating influence of MA on the relation between ANS and math performance. Given the different causal interpretations proposed as an explanation of the association between ANS and math performance, two different models were used in the analyses, the first with ANS and the second with math

⁴ Recent research (Gilmore et al., 2013) has shown that the manipulation of perceptual variables, in order to control for their influence, might be important for the relation between ANS acuity and math performance. We therefore ran separate correlation analyses for the two types of stimuli used in the ANS task. The results showed the relation was significant both for size ($r(77) = -0.28, p = 0.01$) and area ($r(77) = -0.32, p < 0.01$) controlled stimuli. Further, there was no significant difference between the two correlations, $t = -0.36, p = 0.72$. Also, the correlation was of the same approximate strength for all four ratios (-0.24, -0.29, -0.25, and -0.22 for the 4:3, 5:6, 7:8, and 9:10 ratio, respectively).

Table 2

Partial correlations, pr_k , between (i) Math performance, (ii) Math anxiety and (iii) ANS acuity, and other measures when controlling for the remaining k variables. Significant correlations are displayed in bold.

Variable	pr_k	t	p
<i>(i) Math performance</i>			
ANS acuity	0.05	0.4	0.700
Math anxiety	-0.35	3.1	0.003
General intelligence	0.33	2.9	0.005
Trait anxiety	-0.01	0.1	0.953
State anxiety	0.24	2.0	0.046
Test anxiety	0.05	0.4	0.664
Number scanning speed	0.20	1.7	0.086
Letter scanning speed	-0.02	0.1	0.898
<i>(ii) Math anxiety</i>			
Math performance	-0.35	3.1	0.003
ANS acuity	-0.27	2.3	0.024
General intelligence	-0.10	0.9	0.38
Trait anxiety	0.27	2.4	0.020
State anxiety	0.27	2.3	0.020
Test anxiety	0.41	3.7	<0.001
Number scanning speed	0.05	0.4	0.67
Letter scanning speed	0.05	0.4	0.70
<i>(iii) ANS acuity</i>			
Math performance	0.05	0.4	0.700
Math anxiety	-0.27	2.3	0.024
General intelligence	0.12	1.0	0.315
Trait anxiety	0.16	1.3	0.191
State anxiety	0.09	0.7	0.466
Test anxiety	-0.05	0.4	0.651
Number scanning speed	-0.00	0.0	0.988
Letter scanning speed	0.09	0.8	0.447

performance as independent variable. Fig. 2A shows the mediation for the first of these models with the path coefficients.

The total effect of ANS on math performance (c) was significant ($\beta_c = 0.31, SE = 0.11, p = 0.006$). There was a significant effect of ANS on MA (Path a, $\beta_a = -0.35, SE = 0.11, p = 0.002$) and of MA on math performance (Path b, $\beta_b = -0.36, SE = 0.11, p = 0.001$), showing that participants higher in MA had less acute ANS and performed more poorly at math. The direct effect of ANS on math performance (Path c') was not statistically significant ($\beta_{c'} = 0.18, SE = 0.11, p = 0.10$) by conventional alpha level 0.05. The confidence interval for the indirect effect (Path ab) based on 5000 bootstrap samples was entirely above zero ($\beta_{ab} = 0.13, 95\% CI [0.051, 0.247]$), indicating that the influence of ANS on math performance was mediated by MA ($Z = 2.3, p = 0.02$). The mediation analysis thus implies strong mediation. This is consistent with the analysis above, where the partial correlation between ANS and math performance is negligible in size after controlling for all covariate factors (Table 2, section i). The results of the corresponding analysis for the second model (see Fig. 2b) are very similar with the total

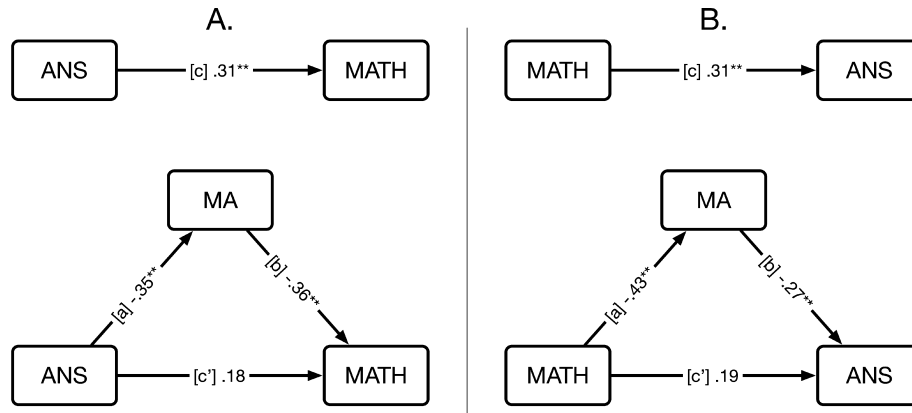


Fig. 2. (A) Path coefficients of the mediation model including ANS acuity as independent variable, math performance (MATH) as dependent variable, and math anxiety (MA) as mediator. (B) Path coefficients of the mediation model including math performance (MATH) as independent variable, ANS acuity (ANS) as dependent variable, and math anxiety (MA) as mediator.

effect of math performance on ANS (Path c, $\beta_c = 0.31$, $SE = 0.11$, $p = 0.006$), the effect of math performance on MA (Path a, $\beta_a = -0.43$, $SE = 0.10$, $p < 0.001$), and the effect of MA on ANS (Path b, $\beta_b = -0.27$, $SE = 0.12$, $p = 0.02$) all being significant while the direct effect of math performance on ANS (Path c') was not ($\beta_{c'} = 0.19$, $SE = 0.12$, $p = 0.10$). Further, as in first model the indirect effect had a confidence interval entirely above zero ($\beta_{ab} = 0.11$, 95% CI [0.018, 0.236]), indicating that the influence of math performance on ANS was mediated by MA ($Z = 1.99$, $p = 0.046$).

4. Discussion

Our results indicate that MA functions as an intermediary factor in the association between ANS acuity and math performance. ANS acuity predicted both math performance and individual differences in math anxiety. Consistent with previous research, MA was negatively associated with math performance. Critically, the association between ANS acuity and MA remained significant after controlling for a battery of potentially confounding cognitive factors and affective components that have been previously related to MA. MA fully accounted for the association between ANS acuity and math performance (see Fig. 2). This study does not allow us to distinguish between the different causal directions proposed, but the analyses indicate full mediation irrespective of causal model. This new finding highlights the importance of an affective factor that has been overlooked in previous research in understanding the relation. Note that our results do not dismiss previous research that has suggested ANS acuity to be a precursor to math performance. Rather, they may nuance this relation and shed light on its underlying mechanisms.

We relied on the use of a dependent measure of math performance consisting of calculation rather than more broad measures such as conceptual math facts. Accordingly, we cannot straightforwardly generalize the conclusions to such dependent measures. Indeed, the “affective drop” on math performance (Ashcraft & Moore, 2009) is particularly pertinent to measures like the timed raw calculation used here. However, given both direct and indirect paths connecting MA and math performance, this relation is likely to be complex. Thus, in a general adult population MA will be indirectly negatively connected to higher conceptual math knowledge merely due to avoidance of educational curricula heavy on math content. This is likely to at least appear developmentally at ages associated with educational levels that allow for personal selection/rejection of course content. It should also be pointed out that our results apply to an adolescent/adult population. Although we

can speculate about the developmental etiology of MA, see discussion below, more research is needed to map out how the relation between ANS and MA emerges as children learn formal mathematics and, accordingly, what the implications of our results are for research that has established a link between ANS acuity in infancy and numerical skills in pre-school children (Starr et al., 2013).

The results are consistent with the hypothesis of Maloney et al. (2010, 2011) that MA influences math performance through a deficiency in basic level number processing. However, our results suggest that this level is more fundamental than previously conceived, generalizing to situations without math or even number symbols. MA is thus apparently related to the individual efficiency of the ancient system for nonverbal number processing (i.e. ANS acuity), which precedes the emergence of symbolic number processing skills both ontogenetically and phylogenetically.

Furthermore, the results suggest that not all kinds of anxiety have detrimental effects on math performance. State-Anxiety was a significant positive predictor of math performance (see Table 2). This type of transient, threat-related anxiety may have positive effects by increased alertness due to rapid onset of physiological arousal responses. The zero order correlation between State-Anxiety and math performance was, however, low and not statistically significant. This indicates that different types of anxiety suppress each other in predicting math performance. Future studies should include anxiety measures other than MA in order to disentangle these effects. The pattern of different signs of the partial correlations between different anxiety types and math performance (Table 2, section i) is in itself a resilient validation of MA as a scientific psychological construct independent of general anxiety and test anxiety.

What are the possible implications of these results for the relations between ANS, math performance, and MA and the developmental etiology of MA? One possibility is that poor math performance both gives rise to higher levels of MA and drives deficits in ANS acuity. We find this interpretation less likely for two reasons. First, previous research (Hembree, 1990; Park, Ramirez, & Beilock, 2014) has indicated that interventions targeted at reducing MA can also improve math performance. In contrast, improving math performance has not been shown to reduce MA (Hembree, 1990). This suggests a relation from MA to math performance, rather than the opposite. Second, recent experimental research investigating the relation between math performance and ANS acuity, on both children and adults, (Lindskog, Winman, & Poom, 2016; Sullivan, Frank, & Barner, 2016) has failed to find support for a causal link by which math proficiency alters ANS acuity.

Our interpretation is instead that having a poor ANS is an individual-specific biological risk factor for later development of MA, which emerges during childhood when the formal number system is first learnt. Children learning the symbolic number system draw on the acuity of their ANS (Gallistel & Gelman, 1992) through shared neural representations. A poor ANS could increase the likelihood of initial failure and negative learning experiences during math education, which in turn could evoke negative affect. On this account, MA develops similarly to a learned specific phobia in a downward spiraling process. Negative environmental triggers during learning induce anxiety, which impedes performance, bringing about more anxiety and avoidance behavior. This developmental etiology of MA fits well both with the hybrid model of MA proposed by Maloney et al. (2011) and the results obtained by Wang et al. (2014) showing that genetic and non-shared environmental influences explain a major amount of the variance in sibling differences in MA. Reliable techniques exist to measure ANS functioning as early as at six months of age (Libertus & Brannon, 2010). Accordingly such methods could be used not only as suggested for early identification of individuals with math learning difficulties, but also those at risk of later developing MA and to tease apart the two possible accounts of the relation between MA, ANS acuity, and math performance.

Supplementary data

The data for this study is available for download at <https://osf.io/xt52t>.

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